A New Boson Realization of Two-Level Pairing Model in Many-Fermion System and its Classical Counterpart

—— *Role of Mixed-Mode Coherent State in the Schwinger Boson Representation for the $su(2) \otimes su(2)$-Algebra* ——

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Abstract

A new method for describing the two-level pairing model in a many-fermion system is presented. Basic idea comes from the Schwinger boson representation for the $su(2) \otimes su(2)$-algebra, in which four kinds of boson operators govern the dynamics. It is well known that in the original fermion space, it is almost impossible to introduce any trial state for the variation except the BCS state in the framework of the mean field approximation. However, in the Schwinger boson representation, various trial states can be defined on the condition that the mean field approximation is useful. In these states, the mixed-mode coherent state is mainly discussed in this paper.

§1. Introduction

The idea of describing the dynamics of many-fermion systems in terms of bosons has a long history and is quite natural, since boson degrees of freedom correspond obviously to canonical variables of classical mechanics. Bosons were already introduced in the collective model of Bohr and Mottelson through the quantization of the oscillations of a liquid drop to describe excitations of nuclei.[1] Sawada derived consistently the so-called random phase approximation in a boson framework having in mind the electron-gas problem.[2] Following his idea, the quasi-particle random phase approximation was formulated to describe collective oscillations in spherical nuclei.[3] With the aid of this method, we can understand the effect of the pairing correlations on the collective oscillations. Afterward, as a natural stream, the so-called boson expansion methods were proposed in various cases.[4)–6)] Including these works, many investigations, which are mainly related to the Lie algebras, were reviewed in Ref. 7). With the aid of these methods, we can estimate deviations from the harmonic oscillations observed in various many-fermion systems. The contribution of Marumori and co-workers including one of the present authors (M. Y.), who established a boson mapping of operators acting on a fermion Hilbert space into operators on a boson Hilbert space,[5] is particularly relevant. Of course, this approach was originally conceived to map a fermion pair operator into a single boson operator. However, the method is very powerful and may be usefully applied under more general conditions. This point was demonstrated by the present authors with Kuriyama.[8]

Correlations play an important role in quantum many-body systems. Their study is a major challenge in various fields of physics. The existence of correlations and the emergence of phase transitions are interdependent. The study of this interdependency in simple models, which has been considered by many authors, is very instructive.[9),10)] In particular, the transition to superconducting or superfluid phase in interacting Fermi systems has been attracting the attention of physicists since Kamerlingh Onnes made his spectacular discovery in 1911. Important aspects of the underlying physics of this phenomena are well described by the two-level pairing model, which has been extensively investigated by various researchers, for instance, in Refs.11) $\sim$ 13), we can find these works.

It is well known that the building blocks in the two-level pairing model are two kinds of Cooper-pair operators and, including the fermion number operators, they compose the $su(2) \otimes su(2)$-algebra. It is also well known that the BCS theory is a powerful approach for this model and we can describe the model in terms of the mean field approximation, i.e., the quasi-particle in the Bogoliubov transformation. In the framework of this theory, we can find the phase transition point to the superconducting phase. However, in many-body systems

with finite degrees of freedom such as nuclei, we cannot observe sharp phase transition. For this problem, the BCS theory may be not so powerful as expected, but, in the framewrok of the mean field approximation in the fermion space, we cannot find an alternative approach. Therefore, if we intend to describe this problem, inevitably, we must go beyond the mean field approximation.

The promising posibility of performing the bosonization of fermion operators by means of a Schwinger-type mapping has also been persued by some autheor.[14),15)] Here, following ideas introduced in previous papers of the present authors with Kuriyama,[16)] we focus on the two-level pairing model in the framewrok of the Schwinger boson representation, based on the use of four kinds of boson operators. Observing that the two-level pairing model is essentially governed by the $su(2) \otimes su(2)$-algebra and the single $su(2)$-algebra can be represented by two kinds of boson operators, it is quite natural to describe the two-level pairing model in terms of four kinds of bosons.

The aim of this paper is to present a new method for describing the two-level pairing model in the Schwinger boson representation. The basic idea is similar to that of the BCS theory: Introduction of the trial state for the variation in the framework of the mean field approximation in the Schwinger boson representation. A straightforward transcription from the original fermion representation leads to the Glauber coherent state. However, in the Schwinger boson representation, the product of the $su(2)$-generators can be diversely transformed by the boson commutation relations. Further, we can define, in various forms, the state which corresponds to the state $|0)$. Here, $|0)$ denotes the state which does not contain any Cooper pair. From the above two points, we can construct the trial state for the variation in various forms. In this paper, we focus on one form, which we will call the mixed-mode coherent state. With the use of this state, we can construct a classical counterpart of the two-level pairing model parametrized in terms of classical canonical variables. The basic idea is borrowed from Ref.8). On the other hand, we pay an attention to the fact that our present model contains three constants of motion. On the basis of this fact, we can derive a disguised form of the two-level pairing model expressed in terms of one kind of boson operator. The basic idea also comes from Ref.8). In the sense of the Dirac quantization rule, both are equivalent to each other. Of course, we will discuss some matters which are closely related to the mixed-mode coherent state.

After recapitulating the two-level pairing model in the fermion space in §2, the model is presented in the framework of the Schwinger boson representation in §3. Three constants of motion are defined and following them, the orthogonal set is presented. In §4, a disguised form of the two-level pairing model is formulated in the framework of one kind of boson operator. The idea comes from the method given in Refs.5) and 8). Section 5 is a highlight

in this paper. After the mixed-mode coherent state is presented, a classical counterpart of the model is formulated. In §6, the correspondence between the original fermion and the Schwinger boson representation is discussed. Finally, an idea how to apply the present method is sketched with some comments.

§2. Two-level pairing model in fermion space

The model discussed in this paper is two-level pairing model in many-fermion system. The two levels are specified by $\sigma = +$ (the upper) and $\sigma = -$ (the lower), respectively. The difference of the single-particle energies between the two levels and the strength of the pairing interaction are denoted as $\hbar\epsilon$ (> 0) and $\hbar^2 G$ (> 0), respectively. The Hamiltonian $\hat{\mathcal{H}}$ is expressed in the form

$$\hat{\mathcal{H}} = \epsilon(\hat{\mathcal{S}}_0(+) - \hat{\mathcal{S}}_0(-)) - G(\hat{\mathcal{S}}_+(+) + \hat{\mathcal{S}}_+(-))(\hat{\mathcal{S}}_-(+) + \hat{\mathcal{S}}_-(-)) \ . \tag{2·1}$$

Here, $\hat{\mathcal{S}}_0(\sigma)$, $\hat{\mathcal{S}}_+(\sigma)$ and $\hat{\mathcal{S}}_-(\sigma)$ are defined as

$$\hat{\mathcal{S}}_0(\sigma) = (\hbar/2)(\hat{\mathcal{N}}_\sigma - \Omega_\sigma) \ , \tag{2·2a}$$

$$\hat{\mathcal{S}}_+(\sigma) = (\hbar/2)\hat{\mathcal{P}}_\sigma^* \ , \qquad \hat{\mathcal{S}}_-(\sigma) = (\hbar/2)\hat{\mathcal{P}}_\sigma \ , \tag{2·2b}$$

$$\hat{\mathcal{N}}_\sigma = \sum_{m=-j_\sigma}^{j_\sigma} \hat{c}_m^*(\sigma)\hat{c}_m(\sigma) \ , \qquad \Omega_\sigma = (2j_\sigma + 1)/2 \ , \tag{2·3a}$$

$$\hat{\mathcal{P}}_\sigma^* = \sum_{m=-j_\sigma}^{j_\sigma} \hat{c}_m^*(\sigma)(-)^{j_\sigma - m}\hat{c}_{-m}^*(\sigma) \ , \ \hat{\mathcal{P}}_\sigma = \sum_{m=-j_\sigma}^{j_\sigma} (-)^{j_\sigma - m}\hat{c}_{-m}(\sigma)\hat{c}_m(\sigma) \ . \tag{2·3b}$$

The set $(\hat{c}_m(\sigma), \hat{c}_m^*(\sigma))$ denotes fermion operator in the single-particle state $(\sigma, j_\sigma, m \ ; j_\sigma =$ half integer, $m = -j_\sigma, -j_\sigma + 1, \cdots, j_\sigma - 1, j_\sigma)$. The fermion number operator and the fermion pairing operator (the Cooper pair) in the single-particle state σ are denoted by $\hat{\mathcal{N}}_\sigma$ and $(\hat{\mathcal{P}}_\sigma, \hat{\mathcal{P}}_\sigma^*)$, respectively. The set $(\hat{\mathcal{S}}_{\pm,0}(\sigma))$ obeys the $su(2)$-algebra, and then, essentially, the presented model is governed by the $su(2)\otimes su(2)$-algebra. The addition of the two sets also obeys the $su(2)$-algebra:

$$\hat{\mathcal{S}}_{\pm,0} = \hat{\mathcal{S}}_{\pm,0}(+) + \hat{\mathcal{S}}_{\pm,0}(-) \ . \tag{2·4}$$

The Hamiltonian (2·1) has three constants of motion. This can be shown from the following relation:

$$[\ \hat{\boldsymbol{\mathcal{S}}}(+)^2 \ , \ \hat{\mathcal{H}}\ ] = [\ \hat{\boldsymbol{\mathcal{S}}}(-)^2 \ , \ \hat{\mathcal{H}}\ ] = [\ \hat{\mathcal{S}}_0 \ , \ \hat{\mathcal{H}}\ ] = 0 \ . \tag{2·5}$$

Here, $\hat{\boldsymbol{\mathcal{S}}}(\sigma)^2$ denotes the Casimir operator for $(\hat{\mathcal{S}}_{\pm,0}(\sigma))$:

$$\hat{\boldsymbol{\mathcal{S}}}(\sigma)^2 = \hat{\mathcal{S}}_0(\sigma)^2 + (1/2)\left[\hat{\mathcal{S}}_-(\sigma)\hat{\mathcal{S}}_+(\sigma) + \hat{\mathcal{S}}_+(\sigma)\hat{\mathcal{S}}_-(\sigma)\right] . \tag{2·6}$$

Since $\hat{\mathcal{H}}$ contains the term $(\hat{\mathcal{S}}_0(+)-\hat{\mathcal{S}}_0(-))$, the Casimir operator for $(\hat{\mathcal{S}}_{\pm,0})$ does not commute with $\hat{\mathcal{H}}$. If the seniority numbers for the two single-particle states are zero, we have

$$\text{the eigenvalue of } \hat{\boldsymbol{\mathcal{S}}}(\sigma)^2 = (\hbar\Omega_\sigma/2)\,(\hbar\Omega_\sigma/2+\hbar) . \tag{2·7}$$

The vacuum $|0)$, which obeys $\hat{c}_m(\sigma)|0) = 0$, is the eigenstate of $\hat{\boldsymbol{\mathcal{S}}}(\sigma)^2$ with the eigenvalue (2·7), and of course, it obeys $\hat{\mathcal{S}}_-(\sigma)|0) = 0$ and $\hat{\mathcal{S}}_0(\sigma)|0) = -(\hbar\Omega_\sigma/2)|0)$. Then, we denote $|0)$ as

$$|0) = |\Omega_+, \Omega_-) . \tag{2·8}$$

The eigenstate of $\hat{\boldsymbol{\mathcal{S}}}(\sigma)^2$ and $\hat{\mathcal{S}}_0$ with arbitrary normalization is expressed in the following form:

$$|(\Omega_+\Omega_-\nu);\kappa) = \left(\hat{\mathcal{S}}_+(+)\right)^{\kappa/2}\left(\hat{\mathcal{S}}_+(-)\right)^{(\nu-\kappa)/2}|\Omega_+, \Omega_-) . \tag{2·9}$$

The state $|(\Omega_+\Omega_-\nu);\kappa)$ satisfies

$$\hat{\mathcal{S}}_0|(\Omega_+\Omega_-\nu);\kappa) = (\hbar/2)(\nu-(\Omega_+ + \Omega_-))|(\Omega_+\Omega_-\nu);\kappa) , \tag{2·10a}$$

$$\hat{\mathcal{N}}_+|(\Omega_+\Omega_-\nu);\kappa) = \kappa|(\Omega_+\Omega_-\nu);\kappa) . \tag{2·10b}$$

Of course, ν and κ denote total fermion number and fermion number in the state $\sigma = +$, respectively, and they are even integers. The quantity κ obeys the condition

$$\begin{aligned}
&\text{(i) } 0 \leq \kappa \leq \nu , \qquad \text{if } \nu \leq 2\mathrm{Min}(\Omega_+, \Omega_-) ,\\
&\text{(ii) } 0 \leq \kappa \leq 2\Omega_+ , \qquad \text{if } 2\Omega_+ \leq \nu \leq 2\Omega_- ,\\
&\text{(iii) } \nu - 2\Omega_- \leq \kappa \leq \nu , \qquad \text{if } 2\Omega_- \leq \nu \leq 2\Omega_+ ,\\
&\text{(iv) } \nu - 2\Omega_- \leq \kappa \leq 2\Omega_+, \qquad \text{if } 2\mathrm{Max}(\Omega_+, \Omega_-) \leq \nu \leq 2(\Omega_+ + \Omega_-) .
\end{aligned} \tag{2·11}$$

Here, if $\Omega_\sigma \geq \Omega_{-\sigma}$, $\mathrm{Min}(\Omega_+, \Omega_-) = \Omega_{-\sigma}$ and $\mathrm{Max}(\Omega_+, \Omega_-) = \Omega_\sigma$. The condition (2·11) can be derived from the inequalities $0 \leq \kappa \leq 2\Omega_+$ and $0 \leq \nu - \kappa \leq 2\Omega_-$, by which the state (2·9) is governed. Then, we can diagonalize $\hat{\mathcal{H}}$ in terms of a superposition of the set $\{|(\Omega_+\Omega_-\nu);\kappa)\}$ with a fixed value of $(\Omega_+\Omega_-\nu)$. In the case which obeys the condition (i) or (ii), the eigenstate (2·9) can be rewritten as

$$\begin{aligned}
&|(\Omega_+\Omega_-\nu);\kappa) = \left(\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)\right)^{\kappa/2}\cdot\left(\hat{\mathcal{S}}_+(-)\right)^{\nu/2}|(-)\Omega_+, \Omega_-) ,\\
&|(-)\Omega_+, \Omega_-) = |\Omega_+, \Omega_-) .
\end{aligned} \tag{2·12}$$

Here, the normalization is arbitrary. Clearly, we have

$$\left(\hat{\mathcal{S}}_+(-)\hat{\mathcal{S}}_-(+)\right)\cdot\left(\hat{\mathcal{S}}_+(-)\right)^{\nu/2}|(-)\Omega_+,\Omega_-)=0\ . \qquad (2\cdot13)$$

The relation (2·13) tells us that in the state $(\hat{\mathcal{S}}_+(-))^{\nu/2}|(-)\Omega_+,\Omega_-)$ all fermions are occupied in the lower level and by the successive operation of $(\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-))$, we can construct the states in which arbitrary numbers of fermions are occupied in the upper level. This means that the operator $(\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-))$ plays the same role as that of the raising operators in the $su(2)$-algebra. In the case of the condition (iii) or (iv), the eigenstate (2·9) is rewritten as

$$\begin{aligned}
&|(\Omega_+\Omega_-\nu);\kappa)=\left(\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)\right)^{\kappa/2-(\nu/2-\Omega_-)}\cdot\left(\hat{\mathcal{S}}_+(+)\right)^{\nu/2-\Omega_-}|(+)\Omega_+,\Omega_-)\ ,\\
&|(+)\Omega_+,\Omega_-)=\left(\hat{\mathcal{S}}_+(-)\right)^{\Omega_-}|\Omega_+,\Omega_-)\ .
\end{aligned} \qquad (2\cdot14)$$

Here, the normalization is arbitrary. We have the relation

$$\left(\hat{\mathcal{S}}_+(-)\hat{\mathcal{S}}_-(+)\right)\cdot\left(\hat{\mathcal{S}}_+(+)\right)^{\nu/2-\Omega_-}|(+)\Omega_+,\Omega_-)=0\ . \qquad (2\cdot15)$$

This is justified from the relation $\hat{\mathcal{S}}_+(-)|(+)\Omega_+,\Omega_-)=(\hat{\mathcal{S}}_+(-))^{\Omega_-+1}|\Omega_+,\Omega_-)=0$, because of the Pauli principle. The state (2·14) is for $\nu\geq 2\Omega_-$, and then, the lower state is fully occupied and the remaining fermions $(\nu-2\Omega_-)$ are occupied in the upper level. The other interpretation of the state (2·14) is in parallel to that of the state (2·12).

The form (2·9) suggests us to introduce the following coherent state:

$$|c_0)=N_c\exp(\alpha_+\hat{\mathcal{S}}_+(+))\exp(\alpha_-\hat{\mathcal{S}}_+(-))|\Omega_+,\Omega_-)\ . \qquad (2\cdot16)$$

Here, N_c denotes the normalization constant and α_σ is complex parameter. The state (2·16) is identical with the trial state for the variation used in the BCS theory. The state $|c_0)$ is constructed in terms of an appropriate superposition of the states with a fixed value of (Ω_+,Ω_-) and different value of ν, i.e., fermion number non-conservation. For the state (2·16), we can introduce the BCS-Bogoliubov transformation and the mean field approximation is applicable. In addition to the state (2·16), we can introduce two coherent states, which are suggested by the states (2·12) and (2·14):

$$|c_-)=N_c\exp[\beta\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)]\exp(\gamma\hat{\mathcal{S}}_+(-))|(-)\Omega_+,\Omega_-)\ , \qquad (2\cdot17\text{a})$$

$$|c_+)=N_c\exp[\beta\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)]\exp(\gamma\hat{\mathcal{S}}_+(+))|(+)\Omega_+,\Omega_-)\ . \qquad (2\cdot17\text{b})$$

Here, also, N_c and (β,γ) denote the normalization constant and complex parameter, respectively. The states (2·17a) and (2·17b) are also composed of the states with a fixed value of

(Ω_+, Ω_-) and different value of ν. The states (2·17a) and (2·17b) may be expected to induce results different from those of BCS theory, i.e., the form (2·16). However, it may be impossible to introduce a transformation which makes the mean field approximation applicable as it stands. The main aim of this paper is to give a form which enables us to describe the system in the framework of the mean field approximation.

§3. The Schwinger boson representation for the present system

Instead of investigating the two-level pairing model in many-fermion system in the original fermion space, we describe this model in boson space in which the Schwinger boson representation is formulated. In §6, we show the connection between both forms. First, we prepare four kinds of boson operators: $(\hat{a}_\sigma, \hat{a}_\sigma^*)$ and $(\hat{b}_\sigma, \hat{b}_\sigma^*)$ $(\sigma = \pm)$. Then, we can define the $su(2)$-algebra in the form

$$\widetilde{S}_+(\sigma) = \hbar \hat{a}_\sigma^* \hat{b}_\sigma \ , \quad \widetilde{S}_-(\sigma) = \hbar \hat{b}_\sigma^* \hat{a}_\sigma \ , \quad \widetilde{S}_0(\sigma) = (\hbar/2)(\hat{a}_\sigma^* \hat{a}_\sigma - \hat{b}_\sigma^* \hat{b}_\sigma) \ . \qquad (\sigma = \pm) \tag{3·1}$$

The Casimir operator $\widetilde{\boldsymbol{S}}(\sigma)^2$, which corresponds to the form (2·6), is expressed as

$$\widetilde{\boldsymbol{S}}(\sigma)^2 = \widetilde{S}(\sigma)(\widetilde{S}(\sigma) + \hbar) \ , \qquad \widetilde{S}(\sigma) = (\hbar/2)(\hat{a}_\sigma^* \hat{a}_\sigma + \hat{b}_\sigma^* \hat{b}_\sigma) \ . \tag{3·2a}$$

Further, we note that the operator $\widetilde{S}_+(+)\widetilde{S}_-(-)$, which corresponds to the raising operator $\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)$, can be re-formed to

$$\widetilde{S}_+(+)\widetilde{S}_-(-) = (\hbar \hat{a}_+^* \hat{b}_+)(\hbar \hat{b}_-^* \hat{a}_-) = \hbar^2 \hat{a}_+^* \hat{b}_-^* \hat{b}_+ \hat{a}_- \ . \tag{3·2b}$$

In the case of $\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-)$, such a re-formation may be impossible.

The Hamiltonian, which corresponds to the form (2·1), can be expressed in the form

$$\widetilde{H} = \epsilon(\widetilde{S}_0(+) - \widetilde{S}_0(-)) - G(\widetilde{S}_+(+) + \widetilde{S}_+(-))(\widetilde{S}_-(+) + \widetilde{S}_-(-)) \ . \tag{3·3a}$$

With the use of $\widetilde{S}_{\pm,0}(\sigma)$ defined in the relation (3·1), the Hamiltonian (3·3a) is written down as

$$\begin{aligned}\widetilde{H} = \ & \epsilon \cdot (\hbar/2)(\hat{a}_+^* \hat{a}_+ - \hat{b}_+^* \hat{b}_+ - \hat{a}_-^* \hat{a}_- + \hat{b}_-^* \hat{b}_-) - G \cdot \hbar^2 (\hat{a}_+^* \hat{a}_+ + \hat{a}_-^* \hat{a}_-) \\ & -G \cdot \hbar^2 (\hat{a}_+^* \hat{a}_+ \hat{b}_+^* \hat{b}_+ + \hat{a}_-^* \hat{a}_- \hat{b}_-^* \hat{b}_-) - G \cdot \hbar^2 (\hat{a}_+^* \hat{b}_-^* \hat{b}_+ \hat{a}_- + \hat{a}_-^* \hat{b}_+^* \hat{b}_- \hat{a}_+) \ .\end{aligned} \tag{3·3b}$$

We can see that the following three operators, which are mutually commuted with each other, are commuted with the Hamiltonian (3·3):

$$\widetilde{L} = (\hbar/2)(\hat{a}_+^* \hat{a}_+ + \hat{b}_+^* \hat{b}_+) \ , \tag{3·4a}$$

$$\widetilde{M} = (\hbar/2)(\hat{a}_+^* \hat{a}_+ + \hat{a}_-^* \hat{a}_-) \ , \tag{3·4b}$$

$$\widetilde{T} = (\hbar/2)(-\hat{a}_+^* \hat{a}_+ + \hat{b}_-^* \hat{b}_- + 1) \ . \tag{3·4c}$$

Further, for $\hbar\hat{a}_+^*\hat{a}_+$, which commutes with $\widetilde{L}$, $\widetilde{M}$ and $\widetilde{T}$, but, does not commute with $\widetilde{H}$, we use the notation

$$\widetilde{K} = \hbar\hat{a}_+^*\hat{a}_+ \ . \tag{3·4d}$$

As is clear from the relation (3·2a), $\widetilde{L}$ denotes the magnitude of the $su(2)$-spin for $\sigma = +$. Of course, it is positive-definite. We can define the $su(1,1)$-algebra in the form

$$\widetilde{T}_+ = \hbar\hat{a}_+^*\hat{b}_-^* \ , \qquad \widetilde{T}_- = \hbar\hat{b}_-\hat{a}_+ \ , \qquad \widetilde{T}_0 = (\hbar/2)(\hat{a}_+^*\hat{a}_+ + \hat{b}_-^*\hat{b}_- + 1) \ . \tag{3·5a}$$

The Casimir operator $\widetilde{\boldsymbol{T}}^2$ can be expressed as

$$\widetilde{\boldsymbol{T}}^2 = \widetilde{T}_0^2 - (1/2)\left[\widetilde{T}_-\widetilde{T}_+ + \widetilde{T}_+\widetilde{T}_-\right] = \widetilde{T}(\widetilde{T} - \hbar) \ . \tag{3·5b}$$

Therefore, $\widetilde{T}$ can be regarded as the magnitude of the $su(1,1)$-spin given in the relation (3·5a). It should be noted that $\widetilde{T}$ is not positive-definite. At the present stage, we cannot give any meanings to $\widetilde{M}$ and $\widetilde{K}$ except the $\hat{a}$-boson number operators. The Hamiltonian (3·3b) can be expressed in the form

$$\begin{aligned}\widetilde{H} = &-\left[\epsilon(\widetilde{L} + \widetilde{M} - (\widetilde{T} - \hbar/2)) + 4G\widetilde{T}\widetilde{M}\right] \\ &+2\left[\epsilon - G(\widetilde{L} + \widetilde{M} - (\widetilde{T} - \hbar/2))\right]\widetilde{K} + 2G\widetilde{K}^2 \\ &-G\cdot\hbar^2\left(\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_- + \hat{a}_-^*\hat{b}_+^*\hat{b}_-\hat{a}_+\right) \ .\end{aligned} \tag{3·6}$$

The Hamiltonian (3·6) can be diagonalized in the space spanned by the orthogonal set $\{|(tml);k\rangle\rangle\}$:

$$\begin{aligned}|(tml);k\rangle\rangle = &\left(\sqrt{k!(2t-1+k)!(2m-k)!(2l-k)!}\right)^{-1} \\ &\times(\hat{a}_+^*)^k(\hat{b}_-^*)^{2t-1+k}(\hat{a}_-^*)^{2m-k}(\hat{b}_+^*)^{2l-k}|0\rangle\rangle \ .\end{aligned} \tag{3·7}$$

Here, $\hbar k$, $\hbar t$, $\hbar m$ and $\hbar l$ denote the eigenvalues of the operators $\widetilde{K}$, $\widetilde{T}$, $\widetilde{M}$ and $\widetilde{L}$, respectively.

For the quantum numbers, k, t, m and l, we note $k = 0, 1, 2, \cdots$, $m = 0, 1/2, 1, \cdots$ and $l = 0, 1/2, 1, \cdots$, but $t = 1/2, 1, 3/2, \cdots$ or $0, -1/2, -1, \cdots$. Further, $2t - 1 + k = 0, 1, 2, \cdots$, $2m - k = 0, 1, 2, \cdots$ and $2l - k = 0, 1, 2, \cdots$. From the above conditions, we obtain

$$l = 0,\ 1/2,\ 1, \cdots, \qquad m = 0,\ 1/2,\ 1, \cdots \ . \tag{3·8}$$

However, concerning t and k, we obtain

$$\text{(i)}\ \ t = 1/2,\ 1,\ 3/2, \cdots\ , \qquad k = 0,\ 1,\ 2, \cdots\ ,\ k^0\ , \tag{3·9a}$$

$$\text{(ii)}\ t = 0,\ -1/2,\ -1, \cdots\ ,\ -(k^0 - 1)/2, \qquad k = 1 - 2t,\ 2 - 2t, \cdots\ ,\ k^0\ , \tag{3·9b}$$

Here, k^0 denotes $k^0 = 2\mathrm{Min}(l, m)$. The diagonalization is performed in the space with a fixed value of (tml), and then, one kind of degree of freedom contributes to the diagonalization. In the next section, we search a method for this procedure in the case (i) shown in the relation (3·9a). At several places, we will contact with the case (ii) briefly in relation to the case (i).

§4. A disguised representation formulated in terms of the MYT boson mapping method

In order to describe the present system in the framework of one kind of degree of freedom, we adopt the basic idea of the MYT boson mapping method, which was presented by Marumori, Yamamura (one of the present authors) and Tokunaga.[5)] First, we prepare a boson space spanned by one kind of boson operator $(\hat{c}, \hat{c}^*)$. The orthogonal set is given by

$$|k\rangle = \left(\sqrt{k!}\right)^{-1} (\hat{c}^*)^k |0\rangle \ . \tag{4·1}$$

Let the state $|k\rangle$ be in one-to-one correspondence with the state $|(tml); k\rangle\!\rangle$ shown in the relation (3·7):

$$|(tml); k\rangle\!\rangle \sim |k\rangle \ . \tag{4·2}$$

Since we investigate the case (i) shown in the relation (3·9a), $|k\rangle$ has its meaning in the case $k \leq k^0$ and we call the space with $k \leq k^0$ as the physical space. Following the basic idea of the MYT boson mapping method, we define the mapping operator $\hat{U}$ from the space $\{|(tml); k\rangle\!\rangle\}$ to the physical space $\{|k\rangle\}$ as follows:

$$\hat{U} = \sum_{k=0}^{k^0} |k\rangle\langle\!\langle (tml); k| \ . \tag{4·3}$$

We can map the state $|(tml); k\rangle\!\rangle$ to $|k\rangle$ by

$$\hat{U}|(tml); k\rangle\!\rangle = |k\rangle \ . \tag{4·4}$$

The operator $\hat{U}$ satisfies

$$\hat{U}^\dagger \hat{U} = \sum_{k=0}^{k^0} |(tml); k\rangle\!\rangle\langle\!\langle (tml); k| = 1 \ , \tag{4·5a}$$

$$\hat{U}\hat{U}^\dagger = \sum_{k=0}^{k^0} |k\rangle\langle k| = \hat{P} \ . \tag{4·5b}$$

Here, $\hat{P}$ plays a role of the projection to the physical space.

Any operator $\widetilde{O}$ working in the space $\{|(tml);k\rangle\!\rangle\}$ can be mapped in the form

$$\hat{O} = \hat{U}\widetilde{O}\hat{U}^\dagger \ . \tag{4·6}$$

For example, we have

$$\hat{U}\widetilde{L}\hat{U}^\dagger = L\hat{P} \ , \qquad \hat{U}\widetilde{M}\hat{U}^\dagger = M\hat{P} \ , \qquad \hat{U}\widetilde{T}\hat{U}^\dagger = T\hat{P} \ , \tag{4·7a}$$

$$L = \hbar l \ , \qquad M = \hbar m \ , \qquad T = \hbar t \ . \tag{4·7b}$$

Further, $\widetilde{K}$ is mapped as

$$\hat{U}\widetilde{K}\hat{U}^\dagger = \hbar\hat{c}^*\hat{c}\hat{P} \ . \tag{4·8}$$

As was discussed by Marshalek,[7] the operator $\hat{P}$ appearing in the expressions (4·7a) and (4·8) may be omitted if restricted to the physical space. Our main interest is in the case $\hbar^2\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-$ appearing in the last term of the Hamiltonian (3·6). In this case, we have

$$\begin{aligned}
&\hat{U}\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-\hat{U}^\dagger \\
&= \sum_{k=0}^{k^0} \sqrt{k+1}\sqrt{2t+k}\sqrt{2m-k}\sqrt{2l-k}\cdot|k+1\rangle\langle k| \\
&= \sum_{k=0}^{k^0} \sqrt{k+1}\sqrt{2t+k}\sqrt{2m-k}\sqrt{2l-k}\cdot\frac{1}{\sqrt{k+1}}\hat{c}^*|k\rangle\langle k| \\
&= \hat{c}^*\cdot\sqrt{2t+\hat{c}^*\hat{c}}\sqrt{2m-\hat{c}^*\hat{c}}\sqrt{2l-\hat{c}^*\hat{c}}\cdot\hat{P} \ .
\end{aligned} \tag{4·9}$$

Then, we obtain

$$\begin{aligned}
&\hat{U}\hbar^2\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-\hat{U}^\dagger \\
&= \sqrt{\hbar}\hat{c}^*\cdot\sqrt{2T+\hbar\hat{c}^*\hat{c}}\sqrt{2M-\hbar\hat{c}^*\hat{c}}\sqrt{2L-\hbar\hat{c}^*\hat{c}}\cdot\hat{P} \ .
\end{aligned} \tag{4·10}$$

We can see that the form (4·10) is a mixture of the Holstein-Primakoff representations of the $su(2)$- and the $su(1,1)$-algebra under the correspondence $\sqrt{\hbar}\hat{a}_+^* \to \sqrt{\hbar}\hat{c}^*$, $\sqrt{\hbar}\hat{b}_-^* \to \sqrt{2T+\hbar\hat{c}^*\hat{c}}$, $\sqrt{\hbar}\hat{a}_- \to \sqrt{2M-\hbar\hat{c}^*\hat{c}}$ and $\sqrt{\hbar}\hat{b}_+ \to \sqrt{2L-\hbar\hat{c}^*\hat{c}}$. In this case, also we may omit $\hat{P}$. Under the above idea, the Hamiltonian (3·6) is mapped to the following form:

$$\begin{aligned}
\hat{H} = &-\left[\epsilon(L+M-(T-\hbar/2))+4GTM\right] \\
&+2\left[\epsilon-G(L+M-(T-\hbar/2))\right]\hat{K}+2G\hat{K}^2 \\
&-G\Big[\sqrt{\hbar}\hat{c}^*\cdot\sqrt{2T+\hat{K}}\sqrt{2M-\hat{K}}\sqrt{2L-\hat{K}} \\
&\qquad +\sqrt{2L-\hat{K}}\sqrt{2M-\hat{K}}\sqrt{2T+\hat{K}}\cdot\sqrt{\hbar}\hat{c}\Big] \ ,
\end{aligned} \tag{4·11}$$

$$\hat{K} = \hbar\hat{c}^*\hat{c} \ . \tag{4·12}$$

We can see that our system can be described in terms of one kind of degree of freedom $(\hat{c}, \hat{c}^*)$ under a fixed value of (T, M, L). The above form is unchanged in the case (ii) shown in the relation (3·9b). It should be noted that in this case T is negative.

Let us replace the operator $(\hat{c}, \hat{c}^*)$, which is the q-number, with the c-number (c, c^*) regarded as canonical:

$$\hat{c} \longrightarrow c \ , \qquad \hat{c}^* \longrightarrow c^* \ . \quad (\hat{K} \longrightarrow K) \tag{4·13}$$

Then, we have

$$\begin{aligned} H = & -[\epsilon(L + M - (T - \hbar/2)) + 4GTM] \\ & +2[\epsilon - G(L + M - (T - \hbar/2))]K + 2GK^2 \\ & -G(\sqrt{\hbar}c^* + \sqrt{\hbar}c)\sqrt{2T + K}\sqrt{2M - K}\sqrt{2L - K} \ . \end{aligned} \tag{4·14}$$

The Hamiltonian (4·14) can be rewritten as

$$\begin{aligned} H = & -[\epsilon(L + M - (T - \hbar/2)) + 4GTM] \\ & +2[\epsilon - G(L + M - (T - \hbar/2))]K + 2GK^2 \\ & -2G\sqrt{K(2T + K)(2M - K)(2L - K)}\cos\psi \ . \end{aligned} \tag{4·15}$$

Here, ψ is defined as

$$\sqrt{\hbar}c = \sqrt{K}e^{-i\psi} \ , \qquad \sqrt{\hbar}c^* = \sqrt{K}e^{i\psi} \ . \tag{4·16}$$

The quantities K and ψ may be regarded as action and angle variables.

It may be self-evident that the Hamiltonian H is the classical counterpart of the Hamiltonian $\hat{H}$, because we have the following relations given below for the commutators and the Poisson brackets: The pioneering idea was presented by Marshalek and Holzwarth.[17)] For any functions $f(x)$, $\xi(x)$ and $\eta(x)$ $(g(x) = \xi(x)\eta(x))$ for x, there exist the relations

$$\begin{aligned} & [\ \sqrt{\hbar}\hat{c}\ , f(\hat{K})\ ] = \hbar \cdot \sqrt{\hbar}\hat{c}f'(\hat{K}, \hbar) \ , \\ & [\ \xi(\hat{K}) \cdot \sqrt{\hbar}\hat{c}\ ,\ \sqrt{\hbar}\hat{c}^* \cdot \eta(\hat{K})\ ] = \hbar \cdot \left[g(\hat{K}) + \hat{K}g'(\hat{K}, \hbar)\right] \ , \end{aligned} \tag{4·17a}$$

$$\begin{aligned} & [\ \sqrt{\hbar}c\ ,\ f(K)\ ]_P = (-i) \cdot \sqrt{\hbar}cf'(K) \ , \\ & [\ \xi(K) \cdot \sqrt{\hbar}c\ ,\ \sqrt{\hbar}c^* \cdot \eta(K)\ ]_P = (-i)[g(K) + Kg'(K)] \ , \end{aligned} \tag{4·17b}$$

Here, $f'(\hat{K}, \hbar)$ and $g'(\hat{K}, \hbar)$ denote the differences of the first order for $f(\hat{K})$ and $g(\hat{K})$ with respect to $\hbar$. The difference for $F(\hat{K})$ is defined as

$$F'(\hat{K}, \hbar) = (F(\hat{K}) - F(\hat{K} - \hbar))/\hbar \ . \tag{4·18}$$

Of course, $f'(K)$ and $g'(K)$ denote the differentials of the first order for $f(K)$ and $g(K)$. The Poisson bracket for A and B is defined as

$$[\ A\ ,\ B\ ]_P = \frac{1}{i\hbar}\left(\frac{\partial A}{\partial c}\frac{\partial B}{\partial c^*} - \frac{\partial B}{\partial c}\frac{\partial A}{\partial c^*}\right) = \frac{\partial A}{\partial \psi}\frac{\partial B}{\partial K} - \frac{\partial B}{\partial \psi}\frac{\partial A}{\partial K}\ . \tag{4·19}$$

In the sense of Dirac, the expressions (4·17) and (4·18) show that the quantum and the classical system correspond to each other. The term, which are of the higher order than $O(\hbar^0)$, appearing on the right-hand sides of the relation (4·17a) automatically disappear in the classical form (4·17b). They express the quantal fluctuations. The term (4·10) which is contained in the Hamiltonian (4·11) is in the above situation.

§5. Mixed-mode coherent state inducing the classical counterpart

In §4, we derived a disguised form of the two-level pairing model in the Schwinger boson representation and its classical counterpart. The basic ideas were the use of the MYT boson mapping method and the replacement of the q-number with the corresponding c-number. In this section, we show that there exists a wave packet, which we call a mixed-mode coherent state, inducing the classical counterpart of the disguised form. The idea is borrowed from that presented by the present authors with Kuriyama and it was applied to the cases of the $su(2)$- and the $su(1,1)$-algebra.[8)]

First, we note the form (3·7). In the case (3·9a), the state (3·7) is rewritten as

$$|(tml);k\rangle\!\rangle = \sqrt{\frac{(2t-1)!}{(2m)!(2l)!}}\sqrt{\frac{(2m-k)!(2l-k)!}{k!(2t-1+k)!}}\cdot(\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-)^k|(-)tml\rangle\!\rangle\ , \tag{5·1a}$$

$$|(-)tml\rangle\!\rangle = \left(\sqrt{(2t-1)!(2m)!(2l)!}\right)^{-1}\cdot(\hat{b}_-^*)^{2t-1}(\hat{a}_-^*)^{2m}(\hat{b}_+^*)^{2l}|0\rangle\!\rangle\ . \tag{5·1b}$$

The state $|(-)tml\rangle\!\rangle$ obeys

$$(\hat{a}_-^*\hat{b}_+^*\hat{b}_-\hat{a}_+)|(-)tml\rangle\!\rangle = 0\ . \tag{5·2}$$

We can see, in the relations (5·1) and (5·2), the following two points: (1) The state $|(-)tml\rangle\!\rangle$ can be regarded as the state similar to the states with the minimum weight in the $su(2)$- and the $su(1,1)$-algebra and (2) $\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-$ plays the same role as that of the raising operators in the $su(2)$- and the $su(1,1)$-algebra. Then, following the idea for constructing wave packets inducing the classical counterparts of the $su(2)$- and the $su(1,1)$-algebra developed by the present authors, we set up a wave packet in the form

$$\begin{aligned}|c_-\rangle\!\rangle = N_c\exp\left(\frac{\hbar}{A_-B_+}\frac{V_+}{U_+}\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-\right)\\ \times\exp\left(\frac{1}{\sqrt{\hbar}}\frac{W_-}{U_+}\hat{b}_-^*\right)\exp\left(\frac{A_-}{\sqrt{\hbar}}\hat{a}_-^*\right)\exp\left(\frac{B_+}{\sqrt{\hbar}}\hat{b}_+^*\right)|0\rangle\!\rangle\ .\end{aligned} \tag{5·3}$$

Here, W_-, V_+, A_- and B_+ denote complex parameters and the parameter U_+ and the normalization constant N_c are given as

$$U_+ = \sqrt{1+|V_+|^2}\ , \tag{5·4}$$

$$N_c = (U_+)^{-1}\exp\left(-\frac{1}{2\hbar}(|W_-|^2+|A_-|^2+|B_+|^2)\right)\ . \tag{5·5}$$

The state $|c_-\rangle\!\rangle$ can be rewritten as

$$|c_-\rangle\!\rangle = N_c\exp\left(\frac{V_+}{U_+}\hat{a}_+^*\hat{b}_-^* + \frac{1}{\sqrt{\hbar}}\frac{W_-}{U_+}\hat{b}_-^*\right)\cdot\exp\left(\frac{A_-}{\sqrt{\hbar}}\hat{a}_-^* + \frac{B_+}{\sqrt{\hbar}}\hat{b}_+^*\right)|0\rangle\!\rangle\ . \tag{5·6}$$

It may be interesting to see that the state $|c_-\rangle\!\rangle$ is the product of the coherent states of the $su(2)$- and the $su(1,1)$-spin system, and then, we call it the mixed-mode coherent state. For the state $|c_-\rangle\!\rangle$, we can introduce the boson operators $(\hat{\alpha}_+,\hat{\alpha}_+^*)$, $(\hat{\beta}_-,\hat{\beta}_-^*)$, $(\hat{\alpha}_-,\hat{\alpha}_-^*)$ and $(\hat{\beta}_+,\hat{\beta}_+^*)$ satisfying the condition

$$\hat{\alpha}_+|c_-\rangle\!\rangle = \hat{\beta}_-|c_-\rangle\!\rangle = \hat{\alpha}_-|c_-\rangle\!\rangle = \hat{\beta}_+|c_-\rangle\!\rangle = 0\ . \tag{5·7a}$$

The relation to the original one is given in the following transformation:

$$\hat{a}_+ = U_+\hat{\alpha}_+ + V_+\hat{\beta}_-^* + \frac{V_+W_-^*}{\sqrt{\hbar}}\ ,\qquad \hat{b}_- = V_+\hat{\alpha}_+^* + U_+\hat{\beta}_- + \frac{U_+W_-^*}{\sqrt{\hbar}}\ ,$$

$$\hat{a}_- = \hat{\alpha}_- + \frac{A_-}{\sqrt{\hbar}}\ ,\qquad \hat{b}_+ = \hat{\beta}_+ + \frac{B_+}{\sqrt{\hbar}}\ . \tag{5·7b}$$

The transformation (5·7b) permits us to apply the mean field approximation to the present form. The aim of this section is to show that the expectation value of $\widetilde{H}$ given in the relation (3·6) for $|c_-\rangle\!\rangle$ becomes the Hamiltonian in the classical counterpart (4·14) or (4·15).

First, we calculate the expectation values of $\hbar\hat{a}_+^*\hat{a}_+$, $\hbar\hat{b}_-^*\hat{b}_-$, $\hbar\hat{a}_-^*\hat{a}_-$ and $\hbar\hat{b}_+^*\hat{b}_+$:

$$\langle\!\langle c_-|\hbar\hat{a}_+^*\hat{a}_+|c_-\rangle\!\rangle = (|W_-|^2+\hbar)|V_+|^2 = K\ , \tag{5·8a}$$

$$\langle\!\langle c_-|\hbar\hat{b}_-^*\hat{b}_-|c_-\rangle\!\rangle = |W_-|^2 + (|W_-|^2+\hbar)|V_+|^2 = 2(T-\hbar/2)+K\ , \tag{5·8b}$$

$$\langle\!\langle c_-|\hbar\hat{a}_-^*\hat{a}_-|c_-\rangle\!\rangle = |A_-|^2 = 2M-K\ , \tag{5·8c}$$

$$\langle\!\langle c_-|\hbar\hat{b}_+^*\hat{b}_+|c_-\rangle\!\rangle = |B_+|^2 = 2L-K\ . \tag{5·8d}$$

Here, K, T, M and L denote the expectation values of $\widetilde{K}$, $\widetilde{T}$, $\widetilde{M}$ and $\widetilde{L}$ defined in the relation (3·4):

$$\langle\!\langle c_-|\widetilde{K}|c_-\rangle\!\rangle = K\ ,\quad \langle\!\langle c_-|\widetilde{T}|c_-\rangle\!\rangle = T\ ,\quad \langle\!\langle c_-|\widetilde{M}|c_-\rangle\!\rangle = M\ ,\quad \langle\!\langle c_-|\widetilde{L}|c_-\rangle\!\rangle = L\ . \tag{5·9}$$

From the relation (5·8), we have

$$|W_-| = \sqrt{2(T-\hbar/2)}\ ,\qquad |V_+| = \sqrt{K/2T}\ ,$$

$$|A_-| = \sqrt{2M-K}\ ,\qquad |B_+| = \sqrt{2L-K}\ . \tag{5·10}$$

Since W_-, V_+, A_- and B_+ are complex, we must determine the phase angles. Then, we regard K, T, M and L as the action variables and we denote their canonically conjugate variables (the angle variables) as ψ, ϕ_T, ϕ_M and ϕ_L, respectively. This statement is supported by the following condition:

$$\begin{aligned}
&\langle\!\langle c_-|i\hbar\partial_\psi|c_-\rangle\!\rangle = K\ , \qquad \langle\!\langle c_-|i\hbar\partial_K|c_-\rangle\!\rangle = 0\ ,\\
&\langle\!\langle c_-|i\hbar\partial_{\phi_T}|c_-\rangle\!\rangle = T-\hbar/2\ , \qquad \langle\!\langle c_-|i\hbar\partial_T|c_-\rangle\!\rangle = 0\ ,\\
&\langle\!\langle c_-|i\hbar\partial_{\phi_M}|c_-\rangle\!\rangle = M\ , \qquad \langle\!\langle c_-|i\hbar\partial_M|c_-\rangle\!\rangle = 0\ ,\\
&\langle\!\langle c_-|i\hbar\partial_{\phi_L}|c_-\rangle\!\rangle = L\ , \qquad \langle\!\langle c_-|i\hbar\partial_L|c_-\rangle\!\rangle = 0\ .
\end{aligned} \tag{5·11}$$

The condition (5·11) was introduced by Marumori, Maskawa, Sakata and Kuriyama for choosing collective degrees of freedom in many-fermion system,[18] and later, the present authors also used extensively in various many-body problems.[8),19] For the state (5·3), we obtain the following formula for any variable z:

$$\begin{aligned}
\langle\!\langle c_-|i\hbar\partial_z|c_-\rangle\!\rangle = (i/2)\Biggl[&(|W_-|^2+\hbar)\left(V_+^*\frac{\partial V_+}{\partial z}-V_+\frac{\partial V_+^*}{\partial z}\right)+\left(W_-^*\frac{\partial W_-}{\partial z}-W_-\frac{\partial W_-^*}{\partial z}\right)\\
&+\left(A_-^*\frac{\partial A_-}{\partial z}-A_-\frac{\partial A_-^*}{\partial z}\right)+\left(B_+^*\frac{\partial B_+}{\partial z}-B_+\frac{\partial B_+^*}{\partial z}\right)\Biggr]\ .
\end{aligned} \tag{5·12}$$

With the aid of the formula (5·12), the condition (5·11) with the relation (5·10) leads to the form

$$\begin{aligned}
&W_- = \sqrt{2(T-\hbar/2)}e^{-i\phi_T/2}\ , \qquad V_+ = \sqrt{K/2T}e^{-i\psi-i\phi_M/2-i\phi_L/2}\ ,\\
&A_- = \sqrt{2M-K}e^{-i\phi_M/2}\ , \qquad B_+ = \sqrt{2L-K}e^{-i\phi_L/2}\ .
\end{aligned} \tag{5·13}$$

The expectation value of $\hbar^2\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-$, which characterizes the Hamiltonian (3·6) is calculated in the form

$$\begin{aligned}
\langle\!\langle c_-|\hbar^2\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-|c_-\rangle\!\rangle &= (|W_-|^2+\hbar)U_+V_+^*A_-B_+\\
&= \sqrt{K(2T+K)(2M-K)(2L-K)}e^{i\psi}\ .
\end{aligned} \tag{5·14}$$

With the use of the relations (5·8a), (5·9) and (5·14), we can calculate $\langle\!\langle c_-|\widetilde{H}|c_-\rangle\!\rangle$, the result of which is reduced to the form (4·15). Thus, we obtain the classical counterpart of $\widetilde{H}$ in terms of calculating the expectation value of $\widetilde{H}$ for the state $|c_-\rangle\!\rangle$.

In the case (3·9b), the state (3·7) can be rewritten in the form

$$|(tml);k\rangle\!\rangle = \sqrt{\frac{(1-2t)!}{(2m-1+2t)!(2l-1+2t)!}}\sqrt{\frac{((k-1+2t)!)^3}{k!(2m-k)!(2l-k)!}}$$

$$\times(\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-)^{k-1+2t}|(+)tml\rangle\!\rangle \ , \tag{5·15a}$$

$$|(+)tml\rangle\!\rangle = \left(\sqrt{(1-2t)!(2m-1+2t)!(2l-1+2t)!}\right)^{-1}$$
$$\times(\hat{a}_+^*)^{1-2t}(\hat{a}_-^*)^{2m-1+2t}(\hat{b}_+^*)^{2l-1+2t}|0\rangle\!\rangle \ . \tag{5·15b}$$

The state $|(+)tml\rangle\!\rangle$ obeys

$$(\hat{a}_-^*\hat{b}_+^*\hat{b}_-\hat{a}_+)|(+)tml\rangle\!\rangle = 0 \ . \tag{5·16}$$

In the same idea as that of the case (3·9a), we can set up the following wave packet:

$$|c_+\rangle\!\rangle = N_c \exp\left(\frac{\hbar}{A_-B_+}\frac{V_-}{U_-}\hat{a}_+^*\hat{b}_-^*\hat{b}_+\hat{a}_-\right)$$
$$\times\exp\left(\frac{1}{\sqrt{\hbar}}\frac{W_+}{U_+}\hat{a}_+^*\right)\exp\left(\frac{A_-}{\sqrt{\hbar}}\hat{a}_-^*\right)\exp\left(\frac{B_+}{\sqrt{\hbar}}\hat{b}_+^*\right)|0\rangle\!\rangle$$
$$= N_c \exp\left(\frac{V_-}{U_-}\hat{a}_+^*\hat{b}_-^* + \frac{1}{\sqrt{\hbar}}\frac{W_+}{U_-}\hat{a}_-^*\right)\cdot\exp\left(\frac{A_-}{\sqrt{\hbar}}\hat{a}_-^* + \frac{B_+}{\sqrt{\hbar}}\hat{b}_+^*\right)|0\rangle\!\rangle \ . \tag{5·17}$$

The interpretation of the notations may be not necessary. In the same manner as that of the case (3·9a), which was previously presented, we can formulate the mixed-mode coherent state $|c_+\rangle\!\rangle$ and the final result is unchanged. In this case, the quantity T is negative.

§6. Correspondence between the fermion and the boson representation

Until the present stage, we describe the two-level pairing model in the Schwinger boson representation independently of the model in many-fermion system. In this section, first, we connect two forms with each other. For this purpose, let us set up the following correspondence:

$$\hat{\boldsymbol{\mathcal{S}}}(+)^2 \sim \widetilde{\boldsymbol{S}}(+)^2 \ , \qquad \hat{\boldsymbol{\mathcal{S}}}(-)^2 \sim \widetilde{\boldsymbol{S}}(-)^2 \ , \tag{6·1a}$$
$$\hat{\mathcal{S}}_0(+) \sim \widetilde{S}_0(+) \ , \qquad \hat{\mathcal{S}}_0(-) \sim \widetilde{S}_0(-) \ . \tag{6·1b}$$

With the aid of the relations (2·7) and (3·2), the correspondence (6·1) leads to

$$\hbar\Omega_+/2 \sim (\hbar/2)(\hat{a}_+^*\hat{a}_+ + \hat{b}_+^*\hat{b}_+) \ ,$$
$$\hbar\Omega_-/2 \sim (\hbar/2)(\hat{a}_-^*\hat{a}_- + \hat{b}_-^*\hat{b}_-) \ , \tag{6·2a}$$
$$\hbar\hat{\mathcal{N}}_+ \sim 2\hbar\hat{a}_+^*\hat{a}_+ \ ,$$
$$\hbar\hat{\mathcal{N}}_- \sim 2\hbar\hat{a}_-^*\hat{a}_- \ . \tag{6·2b}$$

Combining the relation (3·4), we have

$$\hbar\Omega_+/2 \sim \widetilde{L} \ , \tag{6·3a}$$
$$\hbar\hat{\mathcal{N}}/4 \sim \widetilde{M} \ , \tag{6·3b}$$
$$(\hbar/2)(\Omega_- - \hat{\mathcal{N}}/2 + 1) \sim \widetilde{T} \ . \tag{6·3c}$$

Here, $\hat{\mathcal{N}}$ denotes the total fermion number:

$$\hat{\mathcal{N}} = \hat{\mathcal{N}}_+ + \hat{\mathcal{N}}_- \ . \tag{6·4}$$

Of course, the first relation of the form (6·2b) is also important:

$$\hbar\hat{\mathcal{N}}_+/2 \sim \widetilde{K} \ . \tag{6·5}$$

We can learn that $\widetilde{M}$ and $\widetilde{K}$ are closely related to the total and the upper level fermion number, respectively. In the form of the quantum numbers, we have

$$\Omega_+ = 2l \ , \qquad \Omega_- = 2m + 2t - 1 \ , \qquad \nu/2 = 2m \ . \tag{6·6}$$

The vacuum $|\Omega_+, \Omega_-)$ corresponds to the following:

$$\begin{aligned} |\Omega_+, \Omega_-) \sim |\Omega_+, \Omega_-\rangle\!\rangle &= \left(\sqrt{\Omega_+!\Omega_-!}\right)^{-1} (\hat{b}_+^*)^{\Omega_+} (\hat{b}_-^*)^{\Omega_-} |0\rangle\!\rangle \\ &= \left(\sqrt{(2l)!(2m+2t-1)!}\right)^{-1} (\hat{b}_+^*)^{2l} (\hat{b}_-^*)^{2m+2t-1} |0\rangle\!\rangle \ . \end{aligned} \tag{6·7}$$

In the fermion space, the state $|\Omega_+, \Omega_-)$ is uniquely specified. However, in the Schwinger boson representation, the state $|\Omega_+, \Omega_-\rangle\!\rangle$ is constructed by successive operation of $\hat{b}_+^*$ and $\hat{b}_-^*$. Therefore, we can make appropriate superposition of $\{|\Omega_+, \Omega_-\rangle\!\rangle;\ \Omega_+, \Omega_- = 0, 1, 2, \cdots\}$, for example,

$$|v_0\rangle\!\rangle = \exp(\beta_+ \hat{b}_+^*) \exp(\beta_- \hat{b}_-^*) |0\rangle\!\rangle \ . \tag{6·8}$$

Here, β_+ and β_- denote complex parameters. We can see that the state $|v_0\rangle\!\rangle$ satisfies the same relation as that for $|\Omega_+, \Omega_-)$: $\hat{\mathcal{S}}_-(+)|\Omega_+, \Omega_-) = \hat{\mathcal{S}}_-(-)|\Omega_+, \Omega_-) = 0$. Therefore, in spite of boson number non-conservation, we may expect that $|v_0\rangle\!\rangle$ plays the same role as that of $|\Omega_+, \Omega_-)$. It should be noted that $|v_0\rangle\!\rangle$ is a kind of the Glauber coherent state. By operating $\exp(\alpha_+ \widetilde{S}_+(+)) \exp(\alpha_- \widetilde{S}_+(-))$ on the state $|v_0\rangle\!\rangle$, we can set up the state $|c_0\rangle\!\rangle$ as follows:

$$\begin{aligned} |c_0\rangle\!\rangle &= N_c \exp(\alpha_+ \widetilde{S}_+(+)) \exp(\alpha_- \widetilde{S}_+(-)) |v_0\rangle\!\rangle \\ &= N_c \exp[(\hbar\alpha_+\beta_+)\hat{a}_+^* + (\hbar\alpha_-\beta_-)\hat{a}_-^* + \beta_+\hat{b}_+^* + \beta_-\hat{b}_-^*] |0\rangle\!\rangle \ . \end{aligned} \tag{6·9}$$

Here, N_c denotes the normalization constant. The state $|c_0\rangle\!\rangle$ in the Schwinger boson representation is equivalent to the state $|c_0)$ treated in the relation (2·16). It is the Glauber coherent state. The detail will be reported in the succeeding paper.

Next, we investigate the correspondence of the forms (2·17a) and (2·17b). First, it should be noted that the states $|(\mp)\Omega_+, \Omega_-)$ introduced in the relations (2·12) and (2·14) correspond

to

$$|(-)\Omega_+,\Omega_-) \sim |(-)\Omega_+,\Omega_-\rangle\!\rangle = \left(\sqrt{(2l)!(2m+2t-1)!}\right)^{-1}(\hat{b}_+^*)^{2l}(\hat{b}_-^*)^{2m+2t-1}|0\rangle\!\rangle \ , \tag{6·10a}$$

$$|(+)\Omega_+,\Omega_-) \sim |(+)\Omega_+,\Omega_-\rangle\!\rangle = \left(\sqrt{(2l)!(2m+2t-1)!}\right)^{-1}(\hat{b}_+^*)^{2l}(\hat{a}_-^*)^{2m+2t-1}|0\rangle\!\rangle \ . \tag{6·10b}$$

Then, noting the relation $2m = \nu/2$, we have

$$\left(\hat{\mathcal{S}}_+(-)\right)^{\nu/2}|(-)\Omega_+,\Omega_-) \sim \left(\widetilde{S}_+(-)\right)^{2m}|(-)\Omega_+,\Omega_-\rangle\!\rangle = |(-)tml\rangle\!\rangle \ , \tag{6·11a}$$

$$\left(\hat{\mathcal{S}}_+(+)\right)^{\nu/2-\Omega_-}|(+)\Omega_+,\Omega_-) \sim \left(\widetilde{S}_+(+)\right)^{1-2t}|(+)\Omega_+,\Omega_-\rangle\!\rangle = |(+)tml\rangle\!\rangle \ . \tag{6·11b}$$

Here, $|(-)tml\rangle\!\rangle$ and $|(+)tml\rangle\!\rangle$ are given in the relations (5·1b) and (5·15b), respectively. Further, we have

$$\hat{\mathcal{S}}_+(+)\hat{\mathcal{S}}_-(-) \sim \widetilde{S}_+(+)\widetilde{S}_-(-) = \hbar^2\hat{a}_+^*\hat{b}_-^*\cdot\hat{b}_+\hat{a}_- \ . \tag{6·12}$$

Then, we have the correspondence

$$|(\Omega_+\Omega_-\nu);\kappa) \sim |(tml);k\rangle\!\rangle \ , \quad (k=\kappa/2) \tag{6·13a}$$

$$|(\Omega_+\Omega_-\nu);\kappa) \sim |(tml);k\rangle\!\rangle \ . \quad (k-1+2t=\kappa/2-(\nu/2-\Omega_-)) \tag{6·13b}$$

The states $|(\Omega_+\Omega_-\nu);\kappa)$ and $|(tml);k\rangle\!\rangle$ in the relation (6·13a) are defined in the relations (2·12) and (5·1a), respectively. The states $|(\Omega_+\Omega_-\nu);\kappa)$ and $|(tml);k\rangle\!\rangle$ in the relation (6·13b) are also given in the relations (2·14) and (5·15a), respectively. The above correspondence permits us to set up the states which correspond to the states (2·17a) and (2·17b):

$$|c_-\rangle\!\rangle = N_c\exp[\beta\widetilde{S}_+(+)\widetilde{S}_-(-)]\exp(\gamma\widetilde{S}_+(-))|v_-\rangle\!\rangle \ , \tag{6·14a}$$

$$|c_+\rangle\!\rangle = N_c\exp[\beta\widetilde{S}_+(+)\widetilde{S}_-(-)]\exp(\gamma\widetilde{S}_+(+))|v_+\rangle\!\rangle \ . \tag{6·14b}$$

Here, $|v_-\rangle\!\rangle$ and $|v_+\rangle\!\rangle$ are defined as

$$|v_-\rangle\!\rangle = \exp(\beta_+\hat{b}_+^*)\exp(\beta_-\hat{b}_-^*)|0\rangle\!\rangle \ , \qquad (|v_-\rangle\!\rangle = |v_0\rangle\!\rangle) \tag{6·15a}$$

$$|v_+\rangle\!\rangle = \exp(\beta_+\hat{b}_+^*)\exp(\alpha_-\hat{a}_-^*)|0\rangle\!\rangle \ . \tag{6·15b}$$

Here, $|v_0\rangle\!\rangle$ is defined in the relation (6·8). The states (6·14a) and (6·14b) are identical with the states given in the relation (5·6) and (5·17), respectively. Introduction of the states (6·15a) and (6·15b) and the re-form of $\widetilde{S}_+(+)\widetilde{S}_-(-)$ given in the relation (6·12) enable us to present the mixed-mode coherent states (5·6) and (5·17). In the original fermion space, it may be impossible to present the above treatment.

§7. Discussion and concluding remark

Finally, as a discussion, we sketch our idea how to describe our system based on the Hamiltonian (4·11) and its classical counterpart (4·14) or (4·15). The detail will be reported in the succeeding paper. The simplest idea may be to search the minimum point of the energy and the small amplitude oscillation around this point. First, we consider the classical case. The Hamiltonian (4·15) can be symbolically expressed as

$$H = F(K) - f(K) + 2f(K)(\sin\psi/2)^2 \ . \tag{7·1}$$

Since $f(K)$ is positive definite, the minimum point appears at least at the point

$$\psi = 0 \ . \tag{7·2}$$

Then, the minimum point $K = K_0$ can be found at least at the point $K \geq 0$. The function $f(K)$ contains $\sqrt{K(2T+K)}$ and $T \geq \hbar/2$. This means the following: Near the region $K = 0$, the function $(F(K) - f(K))$ is a decreasing function, and then, afterward, becomes increasing. Therefore, we can find the minimum point by the condition that the derivative of $(F(K) - f(K))$ for K is equal to zero. The case of the Glauber coherent state does not show always such behaviors. Under this consideration, $K = K_0$ can be found by the condition

$$F'(K_0) - f'(K_0) = 0 \ . \tag{7·3}$$

Thus, the Hamiltonian (4·15) is approximated as

$$H = F(K_0) - f(K_0) + \frac{1}{2}(F''(K_0) - f''(K_0))(K - K_0)^2 + \frac{1}{2}f(K_0)\psi^2 \ . \tag{7·4}$$

Since $(K - K_0)$ and ψ are canonical, the frequency ω is given by

$$\omega = \sqrt{(F''(K_0) - f''(K_0))f(K_0)} \ . \tag{7·5}$$

We can see that $(F''(K_0) - f''(K_0)) > 0$ and $f(K_0) > 0$, and then, ω does not vanish. In the case of the Glauber coherent state, we know that under certain condition ω vanishes. In §1, we mentioned that, in many-body systems such as nuclei, we cannot observe sharp phase transition. For the confirmation of this point, it may be interesting to investigate the behavior of ω. We will report it in the succeeding paper.

Next, we investigate the above idea in the framework of quantum theory. The Hamiltonian is symbolically expressed as

$$\hat{H} = F(\hat{K}) - f(\hat{K}) - (1/2)\left[\left(\sqrt{\hbar}\hat{c}^* - \sqrt{\hat{K}}\right)\mathring{f}(\hat{K}) + \mathring{f}(\hat{K})\left(\sqrt{\hbar}\hat{c} - \sqrt{\hat{K}}\right)\right] \ ,$$
$$\sqrt{\hat{K}}\,\mathring{f}(\hat{K}) = f(\hat{K}) \ . \tag{7·6}$$

First, we set up the following relation:

$$\sqrt{\hbar}\hat{c} = \sqrt{K_0} - i\sqrt{\hbar}\hat{\gamma} \ , \qquad \sqrt{\hbar}\hat{c}^* = \sqrt{K_0} + i\sqrt{\hbar}\hat{\gamma}^* \ . \tag{7·7}$$

Here, $(\hat{\gamma}, \hat{\gamma}^*)$ denotes the fluctuation around the equilibrium value $\sqrt{K_0}$ and it is boson operator. Our starting assumption is in the condition that $(\sqrt{\hbar}\hat{\gamma}, \sqrt{\hbar}\hat{\gamma}^*)$ is of the order $\hbar^0$, but small. In the framework of the quadratic with respect to the fluctuation, we will treat the system. The operator $(\hat{K} - K_0)$ is expressed in the form

$$\hat{K} - K_0 = \sqrt{K_0} \cdot i\sqrt{\hbar}(\hat{\gamma}^* - \hat{\gamma}) + \hbar\hat{\gamma}^*\hat{\gamma} \ . \tag{7·8}$$

Therefore, under the above assumption, $(\hat{K} - K_0)^2$ and $\sqrt{\hat{K}}$ can be approximated as

$$(\hat{K} - K_0)^2 = K_0[i\sqrt{\hbar}(\hat{\gamma}^* - \hat{\gamma})]^2 \ , \tag{7·9}$$

$$\sqrt{\hat{K}} = \sqrt{K_0} + (1/2)i\sqrt{\hbar}(\hat{\gamma}^* - \hat{\gamma}) + (1/8\sqrt{K_0})[(\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}))^2 - 2\hbar] \ . \tag{7·10}$$

Then, we have

$$\sqrt{\hbar}\hat{c}^* - \sqrt{\hat{K}} = (1/2)i\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}) - (1/8\sqrt{K_0})[(\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}))^2 - 2\hbar] \ . \tag{7·11}$$

With the use of the relation (7·11), the following formula is obtained:

$$\begin{aligned}(1/2)\left(\sqrt{\hbar}\hat{c}^* - \sqrt{\hat{K}}\right) \cdot \overset{\circ}{f}(\hat{K}) = &-(1/16)f(K_0)/K_0 \cdot \left[(\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}))^2 - 2\hbar\right] \\ &+(1/4)\overset{\circ}{f}(K_0) \cdot i\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}) \\ &-(1/4)\overset{\circ}{f}{}'(K_0)\sqrt{K_0} \cdot \hbar(\hat{\gamma}^{*2} - \hat{\gamma}^2 + 1) \ ,\end{aligned} \tag{7·12a}$$

$$\begin{aligned}(1/2)\overset{\circ}{f}(\hat{K}) \cdot \left(\sqrt{\hbar}\hat{c} - \sqrt{\hat{K}}\right) = &-(1/16)f(K_0)/K_0 \cdot \left[(\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}))^2 - 2\hbar\right] \\ &-(1/4)\overset{\circ}{f}(K_0) \cdot i\sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}) \\ &+(1/4)\overset{\circ}{f}{}'(K_0)\sqrt{K_0} \cdot \hbar(\hat{\gamma}^{*2} - \hat{\gamma}^2 - 1) \ .\end{aligned} \tag{7·12b}$$

The relations (7·9) and (7·12) give us the following form for $\hat{H}$:

$$\begin{aligned}\hat{H} = &F(K_0) - f(K_0) + (1/2)(F''(K_0) - f''(K_0))\left[\sqrt{K_0} \cdot i\sqrt{\hbar}(\hat{\gamma}^* - \hat{\gamma})\right]^2 \\ &+(1/2)f(K_0)\left[1/(2\sqrt{K_0}) \cdot \sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma})\right]^2 + (\hbar/2)(f'(K_0) - f(K_0)/K_0) \ .\end{aligned} \tag{7·13}$$

Comparison of the Hamiltonian (7·13) with the classical form (7·4) is interesting. The last term in the c-number part comes from the ordering of the boson operator $(\hat{\gamma}, \hat{\gamma}^*)$. Under the present order of the approximation, $(K - K_0)$ and ψ are quantized in the form

$$\hat{K} - K_0 = \sqrt{K_0} \cdot i\sqrt{\hbar}(\hat{\gamma}^* - \hat{\gamma}) \ , \tag{7·14a}$$

$$\hat{\psi} = (1/2\sqrt{K_0}) \cdot \sqrt{\hbar}(\hat{\gamma}^* + \hat{\gamma}) \ . \tag{7·14b}$$

The form (7·14a) is consistent to the form (7·8) and commutation relation for $(\hat{K}-K_0)$ and $\hat{\psi}$ is given as

$$[\ \hat{K}-K_0\ ,\ \hat{\psi}\ ] = -i\hbar\ . \tag{7·15}$$

Certainly, $\hat{\psi}$ is canonical to $(\hat{K}-K_0)$. Then, the Hamiltonian (7·13) is expressed in the form

$$\hat{H} = F(K_0) - f(K_0) + \hbar\omega\hat{d}^*\hat{d} + (\hbar/2)(f'(K_0) - f(K_0)/K_0 + \omega)\ . \tag{7·16}$$

Here, ω is given in the relation (7·5) and $(\hat{d}, \hat{d}^*)$ denotes the boson operator which is expressed in terms of boson $(\hat{\gamma}, \hat{\gamma}^*)$. The last term in the Hamiltonian (7·16) appears as a result of the quantization and it may be interesting to investigate its behavior.

In this paper, we developed a possible method for describing the two-level pairing model in the framework of the mean field approximation. The idea can be found in the Schwinger boson representation for the $su(2)\otimes su(2)$-algebra and the mixed-mode coherent state plays a central role. In the present framework, we can treat the system in the Glauber coherent state, which is equivalent to the BCS theory in the fermion space. As a concluding remark, we mention third possibility for the coherent state. First, we note the correspondence

$$|(\mathit{\Omega}_+\mathit{\Omega}_-\nu);\kappa) \sim |(tml);k\rangle\!\rangle\ . \tag{7·17}$$

Both states are given in the relations (2·12) and (3·7), respectively. The state $|(tml);k\rangle\!\rangle$ can be expressed, for example, explicitly in the form

$$\begin{aligned}|(tml);k\rangle\!\rangle &= (\hat{a}_+^*)^k(\hat{b}_-^*)^{2t-1+k}(\hat{a}_-^*)^{2m-k}(\hat{b}_+^*)^{2l-k}|0\rangle\!\rangle \\ &= (\hat{a}_+^*\hat{b}_-^*)^k(\hat{a}_-^*\hat{b}_+^*)^{2m-k}(\hat{b}_+^*)^{2(l-m)}(\hat{b}_-^*)^{2t-1}|0\rangle\!\rangle\ . \end{aligned}\tag{7·18}$$

The form (7·18) suggests us the following coherent state:

$$|c^0\rangle\!\rangle = N_c\exp(\alpha_+\hat{a}_+^*\hat{b}_-^*)\exp(\alpha_-\hat{a}_-^*\hat{b}_+^*)|v_0\rangle\!\rangle\ . \tag{7·19}$$

Of course, N_c and $(\alpha_+, \alpha_-, \beta_+, \beta_-)$ denote the normalization constant and complex parameters and $|v_0\rangle\!\rangle$ is defined in the relation (6·8). The state (7·19) is rewritten as

$$|c^0\rangle\!\rangle = N_c\exp(\alpha_+\hat{a}_+^*\hat{b}_-^* + \beta_-\hat{b}_-^*)\exp(\alpha_-\hat{a}_-^*\hat{b}_+^* + \beta_+\hat{b}_+^*)|0\rangle\!\rangle\ . \tag{7·20}$$

The state (7·19) is closely related to the $su(1,1)$-algebra and we call it the squeezed coherent state. From the state (7·20), we obtain the other three forms of the squeezed coherent states under the following replacement:

$$\alpha_+ \longleftrightarrow \beta_-\ , \qquad \alpha_- \longleftrightarrow \beta_+\ , \qquad \hat{a}_+^* \longleftrightarrow \hat{b}_-^*\ , \qquad \hat{a}_-^* \longleftrightarrow \hat{b}_+^*\ . \tag{7·21}$$

In the succeeding paper, we will report the comparative investigation of the Glauber, the mixed-mode and the squeezed coherent state including numerical analysis.

Finally, we will give a comment. Our disguised form of the two-level pairing model is presented in the framework of one kind of boson operator. However, this model can be also formulated in terms of the $su(1,1)\otimes su(1,1)$-algebra and with the use of two kinds of bosons, another disguised form is obtained. The details will be reported as a continuation of the present paper.

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